Mean Reversion of Oil Prices

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Abstract

Managing natural resource projects requires that future costs and revenues be forecasted. Most commodity pricing models are fairly simple, involving a slow, steady increase in base prices while including volatility. When analyzing stock prices, this pattern is commonly referred to as the 'random walk'. Complicating the forecasting process is the fact that many commodities exhibit mean reverting tendencies, where prices may increase or decrease, but tend to revert to a long-term mean. Stock price volatility is measured using the standard deviation of the rate of return of a stock. Commodity price volatility is the same; however, when mean reversion exists, the normal standard deviation will overestimate true volatility. This complicates the pricing of many types of derivatives that are based on commodity prices.

This work investigates the mean reversion tendency of oil prices. Specifically, a 25 year database of West Texas Intermediate daily oil prices is analyzed to determine price volatility, mean reversion speed, and the adjusted volatility that should be applied to today's oil-related projects.

Introduction

Real options analysis is a tool intended to value management flexibility in future decisions. The mathematical foundation of real options is based on financial options. An example of a real option can be illustrated by an oil firm that continues to lease potential development tracts even though development is not currently economic. Paying for the lease (keeping a real option open) can preserve the future opportunity of developing the tract (exercising the real option).

The Black-Scholes model is used to determine the value of options, and many of the related valuation techniques have clearly stated assumptions including the lognormal distribution of cash flows. Many financial models, including Black-Scholes, are based on the heat transfer equation from physics and engineering, stated as some variation of Equation (1), representing geometric Brownian motion (Hull, 2009).

$$dS = \mu S dt + \sigma S dz \qquad (1)$$

where S is the stock or commodity price
μ is the expected rate of return
t is time
σ is the volatility of the stock or commodity price
dz is a Wiener process

While many stocks and some commodities are lognormally distributed with Brownian motion, most real projects are not.

Mean Reversion

It has been well documented that many commodities exhibit Brownian motion with mean reversion (Dixit & Pindyck, 1994; Schwartz, 1997; Al-Harthy, 2007). Examples include crude oil, gold, copper, and electricity. This mean reverting tendency limits both upside and downside price potentials that would normally be forecasted using volatility alone. It has been pointed out that the Black-Scholes pricing model will overprice options where the underlying asset is actually mean reverting (Spar & Schwebach, 1998). Schwartz (1997) described several models for the stochastic behavior of commodity prices involving mean reversion.

Some commodities, including oil and natural gas, exhibit mean reversion along with Brownian motion. Over time, prices revert to a long term mean price. Particularly high oil prices will, in time, fall back to a long term mean, and particularly low prices will rise over time. Schwartz (1997) presented several models that may be used to describe price movements in the presence of mean reversion. His Model 1, shown in Equation (2), is a one-factor model that assumes the logarithm of the spot commodity price S follows a mean reverting process.

$$dS = \kappa(\mu - \ln S)S dt + \sigma S dz \qquad (2)$$

where κ is the speed of reversion

Hafner (2003) demonstrated several models using approximations for use in determining the pricing of options under stochastic volatility and mean reversion. This work included models of adjusting the volatility to take into account the mean reversion nature of a commodity. The resulting adjusted volatility could be used in standard models, including Black-Scholes.

Method

Historic spot prices of West Texas Intermediate crude oil are analyzed. The U.S. Department of Energy publishes daily and weekly spot prices for West Texas Intermediate (WTI) crude oil on their website (www.eia.gov). The database begins in 1986 and continues to the present. This database has been studied for several topics:

- To verify that crude oil prices follow a lognormal distribution
- To determine the volatility of crude oil prices
- To determine the mean reversion speed of crude oil prices
- To determine the variables and constants that pertain to the commodity price model [Equation (2)].

Several studies have estimated the mean reversion speed by modeling futures prices (Schwartz, 1997) or using graphical approaches (Skorodumov, 2008) with different outcomes. Using spot oil prices should be more accurate, and the current information provides a large historical database for analysis. Previous graphical approaches do not work well because volatility (scatter in the data) tends to hide any short-term trends.

Method, continued

Daily spot oil prices are used for the time period of 1986-2011. These prices are shown in Figure 1.

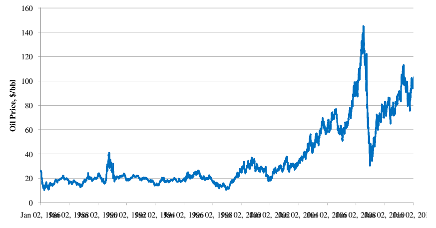

Figure 1. WTI Spot Oil Price, 1986 – 2012.

Results: Distribution

Oil prices are lognormally distributed, as identified in the literature. A histogram of the long-term database is shown in Figure 2, which verifies the lognormal distribution of WTI oil prices. The lognormal fit was verified using @Risk software.

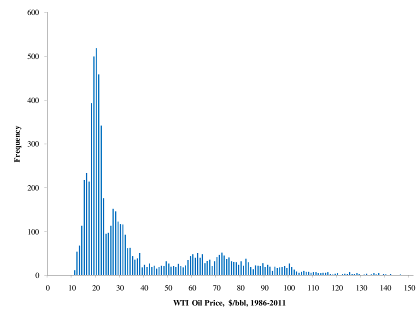

Figure 2. WTI Spot Oil price Histogram, 1986 – 2011

Results: Volatility

Price volatility is the standard deviation of the price rate of return, where this rate of return is defined as Equation (3).

$$r = \ln\left(\frac{S_1}{S_0}\right) \qquad (3)$$

where r is the rate of return
S_0 is the current stock or commodity price
S_1 is the stock or commodity price in time period 1

Because the database is comprised of daily prices, the resulting standard deviation over a time period will be a daily volatility. To convert this to an annual volatility, it must be multiplied by the square root of the number of trading days per year (252 on average), as shown in Equation (4).

$$\sigma_{annual} = \sigma_{daily}\sqrt{T} \cong \sigma_{daily}\sqrt{252} \qquad (4)$$

From the database, we can determine the annual price volatilities, as shown in Table 1. Oil price volatility is not constant. Over the past several years, oil price volatility has been estimated at about 35%, but price speculation in recent years has increased volatility during some periods.

Table 1. Annual Price Volatility, WTI Crude Oil.

Year	Annual Price Volatility, %
2005	35.00
2006	29.04
2007	29.80
2008	62.63
2009	53.44
2010	29.47
2011	34.59

Results: Mean Reversion

Oil prices exhibit mean reversion. The frequency of prices reverting to their long-term mean is not constant, and depends on the time period being studied. The long-term mean also varies depending on the time period. Figures 3 and 4 show the oil price run chart along with the best fitting trend line (drawn using Excel). Figure 3 shows the trend for the period of years 2000 through 2011, with each mean reversion based simply on the graph. This graph shows 13 mean reversions over a period of 12 years, for a mean reversion speed, κ, of 13/12 years or 1.1 year⁻¹.

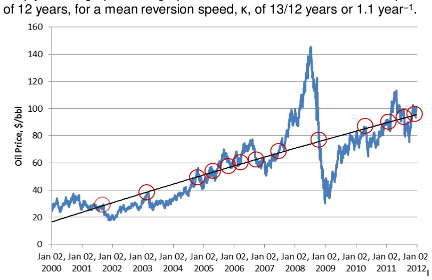

Figure 3. Mean reversion of WTI Crude Oil, 2000-2011.

Results: Mean Reversion

Figure 4 shows the same information, but for the five-year period of 2007-2011. During this period, there were 7 mean reversions, for a mean reversion speed, κ, of 7/5 years or 1.4 year⁻¹. This compares with earlier estimates by Schwartz (1997) of 0.3 to 0.7 using futures prices, and Al-Harthy's (2007) estimate of 0.7. Clearly, the mean-reversion constant is not constant.

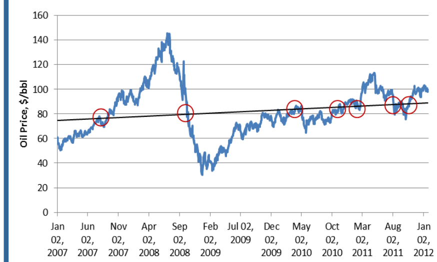

Figure 4. Mean reversion of WTI Crude Oil, 2007-2011

Adjusted Volatility

Hafner (2003) demonstrated that volatility must be adjusted when mean reversion is present. His adjusted volatility is shown in Equation (5).

$$V_{t,T} = \frac{1}{T-t}\int_t^T e^{-2\kappa(T-s)}\sigma_s^2 ds \qquad (5)$$

where

$V_{t,T}$	=	variance with mean reversion
t	=	beginning time period, usually 0
T	=	time horizon
κ	=	mean reversion speed
s	=	incremental time, left as a variable
σ	=	standard deviation (volatility) without mean reversion

The effect of the variables κ and σ are shown in Figure 5. In this graph, the time horizon is two years. This 2 year time period would be the time that an option would mature, or that a real option would be delayed. As an example, if an oil development lease were held for 2 years (a real deferral option having a time horizon of 2 years), with a mean reversion speed of 1.0 and an oil price volatility of 35%, the effective (adjusted) price volatility would be 0.173, half of the original volatility. Because options and other derivative prices are highly dependent on volatility, this would make a dramatic effect on the value of the option.

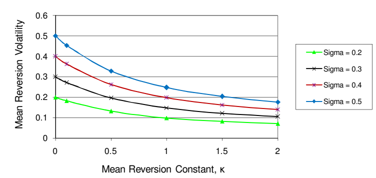

Figure 5. Adjusted Price Volatility, T=2.

Conclusion

Crude oil prices follow Brownian motion with mean reversion, as identified in the literature. West Texas Intermediate crude oil follows a lognormal distribution with slowly changing volatility. The mean reverting speed is not constant, but has shown a cycle length of about 11 months over the past decade, and about 9 months over the past five years. The mean reversion speed is fast enough to have dramatic affects on the adjusted price volatility. The price volatility that should be used for forecasting and derivative analysis is significantly lower than standard methods would indicate.

Many businesses delay oil and gas development on marginal projects until the price increases. When world oil prices increase, a huge number of marginal projects are funded. Unfortunately, most of these projects require several years to complete. Due to the mean reverting nature of prices, by the time the projects are completed, prices will have reverted to their long-term means. It is risky to depend on rising oil prices to justify the development of marginal oil development projects.

The standard methods of determining volatility will over-value derivatives, including real options, on commodities that are mean reverting. Adjusted volatility must be used that incorporate mean reversion to avoid overly optimistic derivative prices.

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